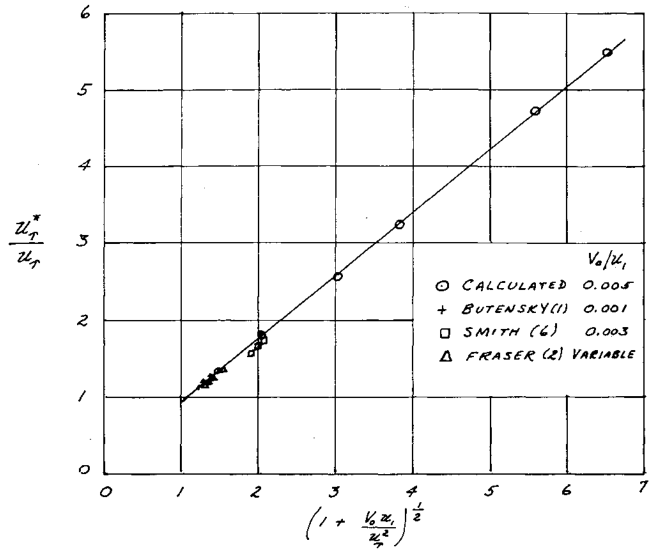

Fig. 3 Relation between $U_\tau{}^*$ and U_τ for transpiration, $dp/dx = 0$.

defect laws represented by Eqs. (1) and (3) are both valid. However, as Figs. 1 and 2 demonstrate, the two correlations are not identical: at the same value of y/δ, the ordinate of Fig. 1 is about 6% greater than the ordinate of Fig. 2. Although the data are subject to uncertainties of this magnitude, the difference between the two correlations is believed significant. The theory behind Eq. (1) is that, for $dp/dx = 0$, the outer portions of transpired and untranspired boundary layers have the same velocity defect law if the shear stress at the inner edge of the outer portion is used to form the scale friction velocity. This shear stress is approximated by the maximum shear stress for positive values of v_0 but is somewhat less than the maximum for the untranspired layer. Data taken in the authors' laboratory for $dp/dx = v_0 = 0$ fall on Fig. 2. If $U_\tau{}^*$ is taken equal to U_τ, the data fall approximately 6% below Fig. 1. However, if $U_\tau{}^*$ is evaluated from the local shear stress at $y/\delta = 0.1$, the data follow Fig. 1.

Stevenson's approach is a major contribution, and for practical calculations is simpler than Eq. (1). However, the implications of Eq. (1) are significant. It is additional evidence of the validity of Clauser's concept that the outer flow of a turbulent boundary layer is a relatively simple region that rides on top of a variable "viscosity" substrate.

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Drag Coefficients of Particles in Gas-Particle Flow

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The use of metallized propellents in rocket engines has resulted in the presence of condensed metal oxides in the form of fine dust particles in the exhaust gas. Since the flow in the nozzle is continuously accelerating, and the exchange of momentum and energy between the gas and solid phases take place in finite time, nonequilibrium phenomena set in. To account for these nonequilibrium phenomena, either in a shock wave[2,3] or in a nozzle,[4] the drag and heat-transfer coefficients of the particles must be known. There has been some evidence[3] that, in a flow past a normal shock, the effect of the uncertainties in the heat-transfer coefficient is not as important as those produced by the drag coefficient.

There have been many measurements of the particle drag coefficients.[1,5] The most recent one was carried out by Rudinger.[5] He found that, for 29 μ particles,

$$C_D = 6000\, Re^{-1.7} \qquad 50 < Re < 300$$

The measurement was carried out in a shock tube; the motion of the particles induced by passage of the shock was recorded by streak photography. This drag coefficient has a much steeper slope with respect to the Reynolds number than the drag coefficients used heretofore. These include the Stokes' formula ($24Re^{-1}$), the data obtained for a steady flow past a sphere ($0.48 + 28\, Re^{-0.85}$), and that measured by Ingebo[6] ($27Re^{-8.4}$). An independent measurement using a method different from any of these cited in the foregoing seems to be desirable.

In this note, we shall propose a steady-state method for the measurement of particle drag coefficient in a supersonic flow of gas-particle stream. Essentially, this method consists of impinging a supersonic stream of gas-particle flow onto a wedge. The relaxation process behind the oblique shock wave will enable us to determine the drag coefficient of the particles. Morgenthaler[7] has used this method to determine the characteristics of the gas-particle flow exhausting from a nozzle.

If we neglect the partial pressure of the solid phase, the equation of motion of the particle phase is

$$\lambda(D\mathbf{q}_s/Dt) = \tfrac{1}{2}C_D\pi r_s{}^2\rho(\mathbf{q} - \mathbf{q}_s)\,|\mathbf{q} - \mathbf{q}_s| \tag{1}$$

in which λ is the mass of the particles, $\mathbf{q}$ is the velocity vector (subscript s refers to the solid phase), C_D is the drag coefficient, ρ is the density of the gas phase, and r_s is the radius of the solid particles.

If we make the reasonable assumption that the particles do not relax when traversing through the shock wave proper, then the initial relative velocity between the two phases right after the shock is normal to the shock wave. Since the force acting between the two phases is in the direction of the relative velocity, then the deceleration of the particle phase and the acceleration of the gas phase will take place in the direction normal to this shock wave. If we take a coordinate system for which x is normal and y is parallel to the shock wave and write the x-component of velocity u, then Eq. (1) becomes

$$\lambda u_s(du_s/dx) = -\tfrac{1}{2}\pi r_s{}^2\rho C_D(u - u_s)^2 \tag{2}$$

By rearrangement,

$$C_D = -\frac{8}{3}\frac{\rho_s}{\rho}\frac{r_s}{[(u_s/u)^2 - 1]}\frac{1}{u_s}\frac{du_s}{dx} \tag{3}$$

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in which ρ_s is the density of the solid particles.

To obtain the quantity $(1/u_s)(du_s/dx)$, we can measure the curvature of the particle streamline $S(x)$ since

$$\frac{1}{u_s}\frac{du_s}{dx} = -\frac{S''(x)}{S'(x)}$$

Another way to obtain $(1/u_s)(du_s/dx)$ is by measuring the particle number density behind the shock n_s. If the number density is measured along a horizontal line x', then

$$\frac{1}{u_s}\frac{du_s}{dx} = -\frac{1}{n_s \sin\theta}\frac{\partial n_s}{\partial x'}$$

where θ is the shock angle. u_s/u is known exactly at the shock. By varying the wedge angle, the drag coefficients can be obtained over a range of relative Reynolds numbers.

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Velocity Profile in the Half-Jet Mixing Region of Turbulent Jets

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A WIDELY used similarity parameter for the nondimensional velocity U/U_0 of jets with exit velocity U_0 is the quantity $\sigma(Y - Y_{0.5})/x$. The x-axis is chosen parallel to the freestream direction or along the axis of an axisymmetric jet.

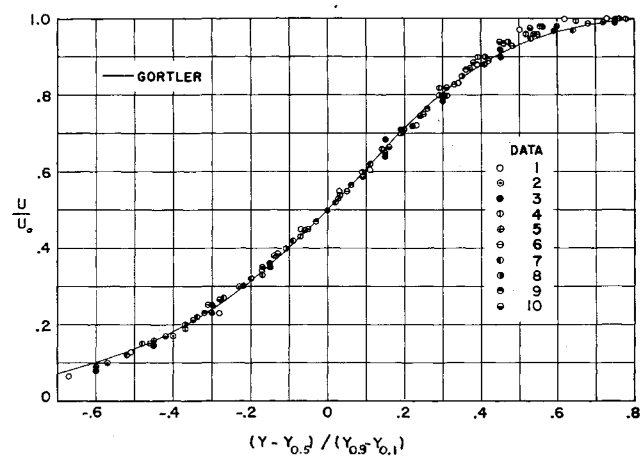

Fig. 1 Dimensionless velocity profile in the half-jet mixing region.

$(Y - Y_{0.5})$ is the lateral distance between point of measurement and the point at which the velocity is half that of the exit velocity, and σ is a constant determined by fitting the measured velocity profile to the theoretical one of either Tollmein or Gortler. The quantity σ is not a universal constant and its value depends on the characteristics of an individual jet. The effect of Mach number on σ has been summed up in Fig. 12 of Ref. 1. See also Refs. 2 and 3. Because of the fairly wide scattering of data, it is hard to give a definite relationship between the two.

However, the variation of σ with different jet parameters (e.g., exit Reynolds number, Mach number, temperature, turbulence level) will be taken care of automatically if one uses the alternate parameter $(Y - Y_{0.5})/(Y_{0.9} - Y_{0.1})$ where $(Y_{0.9} - Y_{0.1})$ is the distance between the points at which the velocity is, respectively, 0.9 and 0.1 of the exit velocity ($U/U_0 = 0.9$ and $U/U_0 = 0.1$). Abramovich[4] used this parameter for the dimensionless velocity profile of his round jet and obtained a profile which fits well with that obtained from results of the plane jet of Albertson et al.[5] Results from our own hot-wire measurements in a 4-in. low-speed round jet also agree well with Abramovich's profile. In view of this good agreement, a few experimental data of other investigators were compiled and replotted for comparison in Table 1 and Fig. 1.

It can be seen that, over a fairly wide range of exit velocities, the different velocity profiles lie very closely along the same curve. This universal curve is best fitted by Gortler's theoretical profile for incompressible jets[6] with his parameter

Table 1 A list of some of the experimental investigations of turbulent jets

Investigation	Date	Nozzle[a] exit shape	Nozzle exit dimensions	Axial distance from exit	Measurement method	Nozzle exit speed
Abramovich[4]	1948	*A*	100 mm	$X/D = 2.5$	...	40 m/sec
Liepmann and Laufer[2]	1947	*R*	60 × 7½ in.	20 cm	Hot wire	59 fps
Laurence[3]	1956	*A*	3½ in.	$X/D = 4$	Pitot pressure	$M = 0.7$
Davies & Fisher[7]	1963	*A*	1 in.	$X/D = 3$	Hot wire	$M = 0.3$
Bradshaw et al.[8]	1963	*A*	2 in.	$X/D = 2$	Hot wire	$M = 0.3$
Maydew and Reed[1]	1963	*A*	3 in.	$X/D = 3$	Pitot pressure	$M = 0.95$
Maydew and Reed[1]	1963	*A*	3 in.	$X/D = 3$	Pitot pressure	$M = 1.49$
Maydew and Reed[1]	1963	*A*	3 in.	$X/D = 3$	Pitot pressure	$M = 1.96$
Present investigation	1964	*A*	4 in.	$X/D = 3$	Hot Wire	140 fps
Present investigation	1964	*A*	4 in.	$X/D = 4$	Hot Wire	140 fps

[a] *A* = axisymmetric, *R* = rectangular.

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